Math 10/11 Enriched: Section 5.2 Solving Rational Functions

- 1. Define what a rational expression is? What is a rational function? How are they different?
- 2. What does it mean to "SOLVE' an equation? What is the difference between "Solving" and "Simplifying"?
- 3. What is an extraneous root when solving a rational equation? Explain:
- 4. Indicate which of the following are rational functions. If not, explain why:

$y = \frac{x+2}{x-3}$	b) $y = x^3 - x^2$	c) $y = \frac{2^{x-3}}{x}$	$d) y = \frac{\sqrt{x}}{x}$
e) $y = x^{-3} + \sqrt{2}x^{-2}$	$f) y = \frac{12}{x}$	g) $y = x^3 + x^2 + x $	h) $x^2 + y^2 + xy = 2$

5. Solve each of the following equations. Indicate any extraneous roots if any:

a) $\frac{4}{x} + \frac{3}{x+2} = 5$	b) $\frac{-2}{x+3} - \frac{5}{x} = 2$

c)
$$\frac{x^2+6}{3} - \frac{7}{2} = \frac{x+15}{2}$$
 d) $\frac{5}{3x-1} - \frac{9}{6x-1} = 2$

e) $\frac{3x}{}$ +	x	2x+3
	x+2	x+2

f)
$$\frac{2x+3}{x+2} - \frac{x+2}{x-1} = \frac{3x}{x-1}$$

g)
$$\frac{3x^2}{x^2-4} - \frac{3x^2}{x^2+5x+6} = \frac{4}{x+3}$$

h)
$$\frac{3x+1}{x^2-2+x} = \frac{2x-3}{x^2-x-6} - \frac{5}{x^2-4x+3}$$

6. Solve each of the following equations. Indicate any extraneous roots if any:

a)
$$\frac{3}{x+2} - \frac{2}{x-1} = 5$$

b)
$$\frac{2}{x+2} + \frac{1}{x} = 1$$

c)
$$\frac{2}{y} = \frac{3}{y^2 + 2}$$

$$\frac{x-2}{x-3} + \frac{x-3}{x-2} = \frac{2x^2}{x^2 - 5x + 6}$$

٥١	$x + \frac{1}{2}$	30	= 3
e,		$\overline{x+8}$	- 5

f)
$$\frac{5}{x+1} + \frac{4}{3} = \frac{x+1}{x-1}$$

g)
$$\frac{2x-1}{2x+1} + \frac{x+1}{x+3} = \frac{3x-1}{2x+1} + \frac{1}{6}$$

h)
$$\frac{2x-3}{x-1} - \frac{x-1}{x+2} = \frac{2x-5}{x+2} + \frac{2-x}{1-x}$$

7. For which value of "x" will $\frac{3+x}{4+x}$ and $\frac{6+x}{8+x}$ be equal?

8. The rational expression $\frac{2x^2+1}{x^2-3}$ may be written as $2+\frac{A}{x^2-3}$, where "A" is an integer. What is the value of "A"?

9. Solve for "k": $\frac{3}{x-1} + \frac{k}{x} + \frac{7}{x+1} = \frac{5x^2 - 4x + 5}{x^3 - x}$

10. Solve for "x": $1 + \frac{1}{1 + \frac{1}{x} + \frac{1}{2x}} = \frac{7}{5}$

11. For what value(s) of "x" is the equation true?
$$\frac{8}{9} = \frac{x}{x + \frac{x}{x + x}}$$

12. Express "k" as a common fraction in terms of "n"
$$\frac{k(n-2)!}{(n+1)!} = \frac{(n-1)!}{(n+2)!}$$

13. What is the greatest integer "n" for which
$$\frac{24n}{n-4}$$
 is an integer? MC COOD2012

14. Solve the following function for "k":
$$\frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}}} = x^k$$

15. For all values other than
$$x=-1$$
 and $x=2$, what is the value of $\frac{2x^2-x}{x^2-x-2}-\frac{4+x}{(x+1)(x-2)}$? AHSME1954

16. Evaluate:
$$\sum_{x=1}^{2003} \frac{1}{x^2 + 7x + 12}$$

- 17. If "a" and "b" are positive integers such that $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 2b}$, find the smallest possible value of a + b:
- 18. If "a" and "b" are positive integers such that $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 2b}$, then which of the following is the smallest possible value of a + b?
 - a) 8
- b) 6
- c) 96
- d) 10
- e) 50

19. Solve for "x" $\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x+1}\right)\left(1 + \frac{1}{x+2}\right)\left(1 + \frac{1}{x+3}\right) = \frac{51}{49}$

20. Challenge: Given that "n" is an integer, for how many values of "n" is $\frac{2n^2-10n-4}{n^2-4n+3}$ an integer? Fermat 2004#25

21. if $\frac{\left(\frac{a}{c} + \frac{a}{b} + 1\right)}{\left(\frac{b}{a} + \frac{b}{c} + 1\right)} = 11$, where "a", "b", and "c" are positive integers, how many different ordered triples of

(a,b,c) are there such that $a+2b+c \le 40$?